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Glueballs, symmetry breaking and axionic strings in non-supersymmetric deformations of the Klebanov-Strassler background

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ABSTRACT: We obtain an analytic solution for a pseudo-scalar massless perturbation of a non-supersymmetric deformation of the warped deformed conifold. This allows us to study D-strings in the infrared limit of non-supersymmetric deformations of the Klebanov-Strassler background. They are interpreted as axionic strings in the dual field theory. Following the arguments of [1], the axion is a massless pseudo-scalar glueball which is present in the supergravity fluctuation spectrum and it is interpreted as the Goldstone boson of the spontaneously broken $\mathrm{U}(1)_B$ baryon number symmetry, being the gauge theory on the baryonic branch.

KEYWORDS: Confinement, Duality in Gauge Field Theories, Brane Dynamics in Gauge Theories, AdS-CFT and dS-CFT Correspondence.

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1. Introduction

After the discovery of the AdS/CFT duality [2] (see also [3]) the natural expectation has been to construct supergravity dual versions of the large N limit of minimally supersymmetric Yang-Mills theories, as well as non supersymmetric ones. In the quest of the holographic dual description of a realistic field theory the findings of the supergravity duals of $\mathcal{N}=1$ SYM theories [4, 5] have certainly been remarkable concrete realizations of this idea. Although in the UV these dual models do not behave like a conventional asymptotically free gauge theory, they are supergravity duals of certain confining $\mathcal{N}=1$ SYM theories. This has led to expect that these two models would fall in the same infrared universality class as the large N limit of the pure $\mathcal{N}=1$ super-gluodynamics. However, as it was very recently shown by Gubser, Herzog and Klebanov [1], it turns out that at least for the Klebanov-Strassler (KS) solution this is not quite the case. In that paper the authors have reached this conclusion by studying a certain perturbation of the RR 2-form field which also mixes with the RR 4-form field.[1] They interpreted D-strings at the bottom of the warped deformed conifold as axionic strings in the dual

[1]Since this particular supergravity perturbation is interpreted as an axion in its dual field theory description, we will call it axionic perturbation even when this is given by a certain combination of perturbations of the RR 2-form and 4-form fields, and it does not include the axion of ten dimensional type IIB supergravity.

$\mathrm{SU}(N+M) \times \mathrm{SU}(N)$ $\mathcal{N}=1$ SYM theory, finding that the axion is a massless pseudo-scalar glueball present in the supergravity fluctuation spectrum. This was interpreted as the Goldstone boson of spontaneously broken $\mathrm{U}(1)_B$ baryon number symmetry. This result provides further evidence for an earlier conjecture that the field theory is on the baryonic branch [6].

On the other hand, non-supersymmetric deformations of the Klebanov-Strassler background have been studied in [7]–[13]. All these non-supersymmetric deformations do not include any perturbation similar to the one discussed in [1]. Hence, it is interesting to investigate the possibility of including axionic perturbations on non-supersymmetric deformations of the Klebanov-Strassler background. Indeed, the motivation of this paper is to obtain an analytic solution for an axionic non-supersymmetric deformation of the KS background, and on this scenario, to show how to extend the results of Gubser, Herzog and Klebanov to non-supersymmetric deformations. We will consider the analytic solution obtained by Kuperstein and Sonnenschein [11] which is a non-singular, non-supersymmetric first order deformation of the KS solution, preserving the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ global symmetry of the KS background. The only non-vanishing vacuum expectation values (VEVs) are those corresponding to operators invariant under this symmetry, i.e. the baryonic operators. Thus, it seems that the theory is on the baryonic branch [6]. Moreover, it would be expected that certain non-supersymmetric deformations of the original background developed some similar properties associated with axionic perturbations as the KS solution itself. In this way, we want to show that in this non-supersymmetric case it is possible to find a massless pseudo-scalar glueball which is present in the supergravity fluctuation spectrum. We present an explicit non-supersymmetric example that provides new evidence in favor of the hypothesis that the $\mathrm{U}(1)_B$ baryon number symmetry is broken by expectation values of baryonic operators. This result agrees with the corresponding one in the case of the supersymmetric KS background reported in [1]. However, there are certain differences due to the fact that superymmetry is not preserved. Interestingly, in breaking supersymmetry there appears a $(0,3)$ form which is related to a non-normalizable mode. Moreover, in the non-supersymmetric deformation of Kuperstein and Sonnenschein [11] there is a second non-normalizable mode related to the deformation of the metric. Remarkably, the supergravity mode associated with the axion that we found turns out to be normalizable, which for this non-supersymmetric deformation allows us to perform a parallel analysis to the one done in [1] for the supersymmetric warped deformed conifold. We will discuss the implications of this axionic perturbation on the non-supersymmetric solution in the dual field theory description.

The paper is organized as follows. We firstly review the main idea of [1] for the supersymmetric KS solution. This is done in section 2. Then, in section 3 we explicitly show how to extend the axionic perturbation ansatz for the analytic non-supersymmetric deformations of the Klebanov-Strassler solution obtained by Kuperstein and Sonnenschein [11]. We explicitly solve the linearized equations for the perturbation ansatz obtaining the general solution. We are able to choose the appropriate boundary conditions to obtain an IR and UV well-behaved solution for the supergravity fluctuation. The non-supersymmetric deformation is controlled by a parameter such that the background becomes the KS solution once the deformation vanishes. In this perturbative approach the axionic perturbation of the non-supersymmetric deformation leads to a normalizable supergravity mode. In section 4 we study some properties of the dual field theories, both supersymmetric and non-supersymmetric ones, when considering the axionic perturbation. Furthermore, we very briefly discuss about the Pando Zayas-Tseytlin background [14] where it was conjectured that the dual field theory would be on the mesonic branch [6]. In section 5 we discuss our results, as well as related open questions that we consider interesting to investigate further.

2. Axionic strings in the KS background

We would like to study the effects of an axionic string in non-supersymmetric deformations of the KS background, and compare them with the case studied in [1] for the supersymmetric warped deformed conifold. In order to do this, in this section we review some of the results of [1] which will be relevant for our purposes. Consider a D1-brane extended in two of the four dimensions of $R^{3,1}$. The D1-brane carries electric charge under the R-R three-form field strength F_3. Hence, one can think that an axion a in four dimensions defined so that

$$*_4 da = \delta F_3 \,, \tag{2.1}$$

experiences monodromy when one makes a loop around the D1-brane. The symbol $*_4$ represents the Hodge dual operation on the 4d Minkowski space-time, while $*$ labels the Hodge dual operation on the 10d background. Note that a very simple ansatz for the axion is $a = a(t) = f_1\, t$, leading to the following ansatz for the perturbation of the RR three-form field strength F_3

$$\delta F_3 = *_4 da = f_1\, dx^1 \wedge dx^2 \wedge dx^3 \,. \tag{2.2}$$

Thus, given the above ansatz for the perturbation, one should add whatever terms are necessary in order to solve the linearized equations of motion. Such a solution would represent a zero-momentum axion. For the warped deformed conifold we will use the notation given in references [15, 16] (see appendix B).

The proposed perturbation ansatz for the KS background given in reference [1] is

$$\begin{aligned}
\delta H_3 &= 0 \,, \\
\delta F_3 &= f_1\, *_4 da + f_2^0(\tau)\, da \wedge dg^5 + \dot{f}_2^0(\tau)\, da \wedge d\tau \wedge g^5 \,, \\
\delta F_5 &= (1 + *)\, \delta F_3 \wedge B_2 \,.
\end{aligned} \tag{2.3}$$

We assume that the perturbations of the rest of the fields are zero. Dot denotes derivative with respect to τ. The axion is assumed to be a function of the (t, x^1, x^2, x^3) coordinates, therefore $d{*_4}da$ vanishes. The sum of the second two terms in the above ansatz for δF_3 leads to an exact two form $-d(f_2^0\, da \wedge g^5)$. We must check that the above perturbation ansatz satisfies the following EOM derived from type IIB supergravity action (see appendix A)

$$d(*F_3) = g_s\, F_5 \wedge H_3 \,, \tag{2.4}$$

$$d(*H_3) = -g_s\, F_5 \wedge F_3 \,, \tag{2.5}$$

together with the Bianchi identities eqs. (A.7), (A.8) and (A.9). The first of the Bianchi equations implies that $d\delta F_3 = 0$, and therefore f_1 is nothing but a constant that is set to 1 [1]. On the other hand, from the second Bianchi identity eq. (A.8) we have $d(\delta F_5) = H_3 \wedge \delta F_3$, which is satisfied for harmonic a. One can show this using the identities (B.20) and (B.21) given in appendix B.

In order to check whether the perturbation ansatz satisfies eq. (2.4), one first obtains the Hodge dual of δF_3 which is given by

$$\begin{aligned} *\delta F_3 =& \, f_1\, h_0^2(\tau)\, \sinh^2\tau\, \frac{\epsilon^4}{96}\, da \wedge d\tau \wedge g^1 \wedge g^2 \wedge g^3 \wedge g^4 \wedge g^5 - \\ & - \frac{f_2^0(\tau)\,\epsilon^{4/3}}{6\,K^2(\tau)}\,(*_4 da) \wedge d\tau \wedge g^5 \wedge dg^5 + \\ & + \frac{3}{8}\,\dot{f}_2^0(\tau)\,\epsilon^{4/3}\,K^4(\tau)\,\sinh^2\tau\,(*_4 da) \wedge g^1 \wedge g^2 \wedge g^3 \wedge g^4 \,. \end{aligned} \tag{2.6}$$

Therefore

$$d(*\delta F_3) = \frac{3\,\epsilon^{4/3}}{4}\left(-\frac{d}{d\tau}\left[\dot{f}_2^0(\tau)\,K^4\,\sinh^2\tau\right] + \frac{8\,f_2^0}{9\,K^2}\right)(*_4 da) \wedge d\tau \wedge \omega_2 \wedge \omega_2\,, \tag{2.7}$$

while

$$\delta F_5 \wedge H_3 = 2\,\frac{f_1}{4}\,(g_s M \alpha')^2\,\frac{d(k(\tau)\,f(\tau))}{d\tau}\,(*_4 da) \wedge d\tau \wedge \omega_2 \wedge \omega_2\,, \tag{2.8}$$

where identities (B.17), (B.19) and eq. (B.15) have been used. Definitions of functions $K(\tau)$, $F(\tau)$, $f(\tau)$ and $k(\tau)$ are given in appendix B. With these expressions, from eq. (2.4) one gets the following second order ordinary differential equation

$$-\frac{d}{d\tau}[\dot{f}_2^0(\tau)\,K^4\,\sinh^2\tau] + \frac{8\,f_2^0}{9\,K^2} = \frac{(g_s M\alpha')^2}{3\,\epsilon^{4/3}}\,(\tau\,\coth\tau - 1)\left(\coth\tau - \frac{\tau}{\sinh^2\tau}\right). \tag{2.9}$$

Now, in order to solve the above ODE one starts with the homogeneous differential equation

$$-\frac{d}{d\tau}[\dot{f}_2^0(\tau)\,K^4\,\sinh^2\tau] + \frac{8\,f_2^0}{9\,K^2} = 0\,, \tag{2.10}$$

which is solved by the functions

$$f_2^{0\,(1)}(\tau) = [\sinh(2\tau) - 2\tau]^{1/3}\,, \tag{2.11}$$

$$f_2^{0\,(2)}(\tau) = [\sinh(2\tau) - 2\tau]^{-2/3}\,. \tag{2.12}$$

The general solution of an inhomogeneous equation of the form

$$\frac{d^2 y(\tau)}{d\tau^2} + p(\tau)\,\frac{dy(\tau)}{d\tau} + q(\tau)\,y(\tau) = j(\tau)\,, \tag{2.13}$$

is

$$y(\tau) = c_1\,y_1(\tau) + c_2\,y_2(\tau) + y_p(\tau)\,, \tag{2.14}$$

where c_1 and c_2 are constants, while y_1 and y_2 are independent solutions of the homogeneous ODE ($f_2^{0\,(1)}$ and $f_2^{0\,(2)}$, respectively), and $y_p(\tau)$ is a particular solution of the inhomogeneous equation given by

$$y_p(\tau) = -y_1(\tau)\int_0^\tau dx\,\frac{y_2(x)}{W(y_1,y_2)(x)}\,j(x) + y_2(\tau)\int_0^\tau dx\,\frac{y_1(x)}{W(y_1,y_2)(x)}\,j(x)\,, \tag{2.15}$$

where the wronskian is defined as usual

$$W(y_1, y_2)(x) = y_1(x)\, y_2'(x) - y_2(x)\, y_1'(x)\,, \tag{2.16}$$

and prime denotes derivative with respect to x. In the present case it is not difficult to obtain the solution

$$f_2^0(\tau) = -2\,\frac{\epsilon^{4/3}}{12}\,\frac{1}{K^2(\tau)\,\sinh^2\tau}\int_0^\tau dx\, h_0(x)\,\sinh^2 x\,. \tag{2.17}$$

Indeed, this solution $f_2^0(\tau)$ behaves like τ for small τ, while it falls as $\tau\, e^{-2/3\,\tau}$ for $\tau \to \infty$. In section 3 we will give the explicit asymptotic expressions. Notice that we have used the label 0 to indicate that $f_2^0(\tau)$ is the function corresponding to the supergravity fluctuation on the supersymmetric warped deformed conifold. This notation will be useful in the next section. The related definitions and conventions are explicitly given in appendix B.

Now, an important point is to show that δF_3 is normalizable. To do so it is necessary to integrate the fluctuation such that the integral of $\sqrt{|g|}\,|\delta F_3|^2$ over τ, where g is the ten dimensional metric, must be finite. In addition, note that

$$\sqrt{|g|}\,|\delta F_3|^2\, d^{10}x = \delta F_3\,\wedge *\delta F_3\,. \tag{2.18}$$

The fact that the above integral reduces to the sum of three single well-behaved integrals at small τ and falls like $e^{-2\tau/3}$ for $\tau \to \infty$, guarantees that the perturbation is normalizable. This is a normalizable zero-mode of the KS background [1]. We will return to the analysis of this perturbation in the next sections, in order to compare it with related issues from non-supersymmetric deformations of the KS solution.

3. Axionic strings in non-supersymmetric deformations of the KS background

In this section, we study axionic strings in non-supersymmetric deformations of the KS background. An analytic non-supersymmetric deformation of the warped deformed conifold has been obtained by Kuperstein and Sonnenschein [11]. Their solution is based on an expansion of the fields in terms of a parameter $\tilde{\delta}$, which accounts for the non-supersymmetric deformation of the background. The explicit deformations were obtained using a modification of the superpotential method introduced in [7]. This will be briefly reviewed in this section in order to have the necessary background to perform our calculations.

The idea is that the axionic perturbation ansatz can be understood as a perturbation of the non-supersymmetric deformations of the KS background. Therefore, at first order in $\tilde{\delta}$ we will have the same EOM, as well as the first order equations derived from the superpotential for the deformed non-supersymmetric background like in [11]. On the other hand, the perturbation of the 2-form and 4-form fields will be controlled by perturbation equations analogous to the ones in the previous section.

In order to show explicitly how it works, let us remember that we deal with the linearized EOM. Therefore, the dilaton EOM leads to the same equation as in the supersymmetric case, i.e. $e^{\phi}\, F_3 \wedge *F_3 = e^{-\phi}\, H_3 \wedge *H_3$. In addition, there is the usual relation

$g_s = e^{\phi}$. The equation for the five-form field strength has the form $dF_5 = H_3 \wedge F_3$. All these equations are satisfied by the non-supersymmetric deformation of Kuperstein and Sonnenschein, since they are satisfied by the first order equations for the deformations. The equations for the perturbation ansatz look similar to the corresponding ones in section 2. We will show this for $f_2(\tau)$ in this section.

Analytic non-supersymmetric deformations of the KS background have been constructed by Kuperstein and Sonnenschein using the superpotential method. This method has been extensively used for studying gravitational RG flows in five-dimensional gauged supergravity [17]–[19] and [7, 11]. Here we briefly review this method following [7, 11]. We use the notation for the warped deformed conifold metric of [15, 16], while for the non-supersymmetric deformation we follow [11]. Let us consider the most general metric with SU(2) $\times$ SU(2) isometry, that includes the Klebanov-Strassler solution by setting $\tilde{\delta} = 0$. We can write the metric using the following ansatz

$$\begin{aligned} ds_{10}^2 = 2^{1/2}\, 3^{3/4} \Big[& e^{-5q(\tau)+2Y(\tau)}\,(-dt^2 + d\vec{x}^2) + \frac{1}{9}\, e^{3q(\tau)-8p(\tau)}\,(d\tau^2 + (g^5)^2) + \\ & + \frac{1}{6}\, e^{3q(\tau)+2p(\tau)+y(\tau)}\,((g^1)^2 + (g^2)^2) + \frac{1}{6}\, e^{3q(\tau)+2p(\tau)-y(\tau)}\,((g^3)^2 + (g^4)^2) \Big] . \end{aligned} \tag{3.1}$$

For definitions of the one forms g^i's we refer the reader to appendix B. We assume the axion as a function $a = a(t, x^1, x^2, x^3)$, while the dilaton is $\phi = \phi(\tau)$. We consider the following ansatz for the fields

$$B_2 = -(\tilde{f}(\tau)\, g^1 \wedge g^2 + \tilde{k}(\tau)\, g^3 \wedge g^4)\,, \tag{3.2}$$

$$F_3 = 2\,P\, g^5 \wedge g^3 \wedge g^4 + d[\tilde{F}(\tau)\,(g^1 \wedge g^3 + g^2 \wedge g^4)]\,, \tag{3.3}$$

$$F_5 = \mathcal{F}_5 + *\mathcal{F}_5\,, \tag{3.4}$$

where

$$\mathcal{F}_5 = -\tilde{L}(\tau)\, g^1 \wedge g^2 \wedge g^3 \wedge g^4 \wedge g^5\,, \tag{3.5}$$

with

$$\tilde{L}(\tau) = Q + \tilde{f}(\tau)\,(2P - \tilde{F}(\tau)) + \tilde{k}(\tau)\,\tilde{F}(\tau)\,, \tag{3.6}$$

while the perturbation ansatz is given by

$$\delta H_3 = 0\,, \tag{3.7}$$

$$\delta F_3 = f_1 *_4 da + f_2(\tau)\, da \wedge dg^5 + \dot{f}_2(\tau)\, da \wedge d\tau \wedge g^5\,, \tag{3.8}$$

$$\delta F_5 = (1 + *)\, \delta F_3 \wedge B_2\,, \tag{3.9}$$

where we have used the rescaled functions as in [11]

$$\tilde{f}(\tau) = -2P\, g_s\, f(\tau)\,, \qquad \tilde{k}(\tau) = -2P\, g_s\, k(\tau)\,, \qquad \tilde{F}(\tau) = 2P\, F(\tau)\,. \tag{3.10}$$

Q and P are constants related to the number of regular and fractional D3-branes, respectively. Indeed, Q is proportional to the number of regular D3-branes, while $P = \frac{1}{4}\, M\, l_s^2$. We use the convention where $Q = 0$ [20, 4]. Starting from the IIB supergravity action (A.1)

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it leads to the following one-dimensional effective action

$$S_{\rm eff} = \frac{2}{\kappa^2} \int \left(-\frac{1}{2} G_{ij}\, \dot{\phi}^i\, \dot{\phi}^j - V(\phi) \right) d\tau \,. \tag{3.11}$$

This effective action is the same as in [11]. As before, dot stands for derivative with respect to τ. For the so-called metric in the effective action above we have the following structure

$$\begin{aligned} G_{ij}(\tau)\, \dot{\phi}^i(\tau)\, \dot{\phi}^j(\tau) \;=\; & e^{4p(\tau)-4q(\tau)+4Y(\tau)} \Big(-18\,\dot{Y}^2 + 45\,\dot{q}^2 + 30\,\dot{p}^2 + \frac{3}{2}\,\dot{y}^2 + \frac{3}{4}\,\dot{\phi}^2 + \\ & + \frac{\sqrt{3}}{2} 3 e^{-\phi(\tau)-6q(\tau)-4p(\tau)} \left(e^{-2y(\tau)}\, \dot{\tilde{f}}^2 + e^{2y(\tau)} \dot{\tilde{k}}^2 \right) + \\ & + 3\sqrt{3}\, e^{\phi(\tau)-6q(\tau)-4p(\tau)}\, \dot{\tilde{F}}^2) \,. \end{aligned} \tag{3.12}$$

The effective potential is given by [11]

$$\begin{aligned} V(\phi) \;=\; e^{4Y(\tau)} \bigg(& \frac{1}{3}\, e^{-16p(\tau)-4q(\tau)} - 2\, e^{-6p(\tau)-4q(\tau)}\, \cosh y + \frac{3}{4}\, e^{4p(\tau)-4q(\tau)}\, \sinh^2 y + \\ & + \frac{3\sqrt{3}}{4}\, e^{\phi(\tau)-10q(\tau)+2y(\tau)}\, (2\,P - \tilde{F})^2 + \frac{3\sqrt{3}}{4}\, e^{\phi(\tau)-10q(\tau)-2y(\tau)}\, \tilde{F}^2 + \\ & + \frac{3\sqrt{3}}{4}\, e^{-\phi(\tau)-10q(\tau)}\, (\tilde{k} - \tilde{f})^2 + \frac{9}{2}\, e^{-4p(\tau)-16q(\tau)}\, \tilde{L}^2 \bigg) \,. \end{aligned} \tag{3.13}$$

The potential $V(\phi)$ can be derived from the following superpotential

$$W = -3\, e^{4Y(\tau)+4p(\tau)-4q(\tau)}\, \cosh y - 2\, e^{4Y(\tau)-6p(\tau)-4q(\tau)} - 3\sqrt{3}\, e^{4Y(\tau)-10q(\tau)}\, \tilde{L}\,, \tag{3.14}$$

where

$$V(\phi) = \frac{1}{8}\, G^{ij}\, \frac{\partial W}{\partial \phi^i} \frac{\partial W}{\partial \phi^j}\,, \tag{3.15}$$

where $\phi^i = (\phi, Y, q, p, y, \tilde{f}, \tilde{k}, \tilde{F})$. In addition, there is a zero-energy condition

$$\frac{1}{2}\, G_{ij}\, \dot{\phi}^i\, \dot{\phi}^j - V(\phi) = 0\,. \tag{3.16}$$

Therefore, the problem reduces to the finding of a non-supersymmetric deformation where the axionic perturbation can be treated as a perturbation of the non-supersymmetric deformation. This is correct since the ansatze for all the fields including the axionic perturbation satisfy the linearized equations of motion of type IIB supergravity.

Now, we show how to implement the perturbation method introduced by Borokhov and Gubser, following the notation of Kuperstein and Sonnenschein. Consider the effective one-dimensional lagrangian

$$L = -\frac{1}{2}\, G_{ij}\, \dot{\phi}^i\, \dot{\phi}^j - V(\phi)\,. \tag{3.17}$$

The new idea introduced in [7] consists in using the superpotential (3.14) to derive solutions satisfying the second order equations (EOM) but not the first order ones. It means that in this case the first order equation

$$\frac{d\phi^i}{d\tau} = \frac{1}{2}\, G^{ij}\, \frac{\partial W}{\partial \phi^j}\,, \tag{3.18}$$

is no longer valid. Now, consider a deformation of the supersymmetric solution $\phi_0^i(\tau)$ written as

$$\phi^i(\tau) = \phi_0^i(\tau) + \tilde{\delta} \cdot \bar{\phi}^i(\tau) + \mathcal{O}(\tilde{\delta}^2)\,, \tag{3.19}$$

such that $\phi_0^i(\tau)$ does satisfy eq. (3.18), where $\tilde{\delta}$ is a small positive constant that controls the non-supersymmetric deformation. It is conventional to introduce the functions

$$\zeta_i = G_{ij}(\phi_0)\left(\frac{d\bar{\phi}^j}{d\tau} - N_k^j(\phi_0)\,\bar{\phi}^k\right), \tag{3.20}$$

where

$$N_k^j(\phi_0) = \frac{1}{2}\frac{\partial}{\partial\phi^k}\left(G^{jl}(\phi_0)\,\frac{\partial W}{\partial\phi^l}\right). \tag{3.21}$$

From the second order equations derived from the lagrangian (3.17), using the expansion (3.19) it is easy to obtain the first order differential equation

$$\frac{d\zeta_i}{d\tau} = -N_i^j(\phi_0)\,\zeta_j\,. \tag{3.22}$$

In addition, from the definition of ζ_i we have

$$\frac{d\bar{\phi}^i}{d\tau} = N_j^i(\phi_0)\,\bar{\phi}^j + G^{ij}(\phi_0)\,\zeta_j\,, \tag{3.23}$$

while the zero-energy condition becomes

$$\zeta_i\,\frac{d\bar{\phi}^i}{d\tau} = 0\,. \tag{3.24}$$

Therefore, the following first order differential equations are obtained

$$\dot{\zeta}_Y = 0\,, \tag{3.25}$$

$$\dot{\zeta}_p = \frac{4\sqrt{3}}{3}\,e^{-4p_0(\tau)-6q_0(\tau)}\,\tilde{L}_0\,(\zeta_q+\zeta_Y) + e^{-10p_0(\tau)}\left(\frac{20}{9}\,\zeta_Y + \frac{8}{9}\,\zeta_q + 2\,\zeta_p\right), \tag{3.26}$$

$$\dot{\zeta}_q = 2\sqrt{3}\,e^{-4p_0(\tau)-6q_0(\tau)}\,\tilde{L}_0\,(\zeta_q+\zeta_Y)\,, \tag{3.27}$$

$$\begin{aligned}\dot{\zeta}_y = &-\left(\frac{1}{3}\,\zeta_Y + \frac{2}{15}\,\zeta_q - \frac{1}{5}\,\zeta_p\right)\sinh y_0 + \zeta_y\,\cosh y_0 - \\ &- 2e^{\phi_0(\tau)+2y_0(\tau)}\,(\tilde{F}_0 - 2\,P)\,\zeta_{\tilde{f}} - 2\,e^{\phi_0(\tau)-2y_0(\tau)}\,\tilde{F}_0\,\zeta_{\tilde{k}}\,,\end{aligned} \tag{3.28}$$

$$\dot{\zeta}_{\tilde{f}+\tilde{k}} = -\frac{2\sqrt{3}}{3}\,P\,e^{-4p_0(\tau)-6q_0(\tau)}\,(\zeta_Y+\zeta_q)\,, \tag{3.29}$$

$$\dot{\zeta}_{\tilde{f}-\tilde{k}} = -e^{-\phi_0(\tau)}\,\zeta_{\tilde{F}} + \frac{2\sqrt{3}}{3}\,e^{-4p_0(\tau)-6q_0(\tau)}\,(\tilde{F}_0 - P)\,(\zeta_q+\zeta_Y)\,, \tag{3.30}$$

$$\begin{aligned}\dot{\zeta}_{\tilde{F}} = &-\frac{1}{\sqrt{3}}\,(\tilde{k}_0 - \tilde{f}_0)\,e^{-4p_0(\tau)-6q_0(\tau)}\,(\zeta_q+\zeta_Y) - \\ &- e^{\phi_0}\left(\cosh(2y_0)\,\zeta_{\tilde{f}-\tilde{k}} + \sinh(2y_0)\,\zeta_{\tilde{f}+\tilde{k}}\right),\end{aligned} \tag{3.31}$$

$$\dot{\zeta}_\phi = (2P - \tilde{F}_0)\,e^{\phi_0(\tau)+2y_0(\tau)}\,\zeta_{\tilde{f}} + \tilde{F}_0\,e^{\phi_0(\tau)-2y_0(\tau)}\,\zeta_{\tilde{k}} - \frac{\tilde{k}_0 - \tilde{f}_0}{2}\,e^{-\phi_0(\tau)}\,\zeta_{\tilde{F}}\,. \tag{3.32}$$

Notice that setting $g_s = e^{\phi} = 1$, the equations above become the ones in reference [11]. We have used the definitions $\zeta_{\tilde{f}\pm\tilde{k}} = \zeta_{\tilde{f}} \pm \zeta_{\tilde{k}}$.

Now, following [11] we show how to solve these first order differential equations when $\zeta_Y = \zeta_p = \zeta_q = 0$. Therefore, from eq. (3.29) $\zeta_{\tilde{f}+\tilde{k}} = X$ where X is a constant of integration. From eq. (3.30) $\dot{\zeta}_{\tilde{f}-\tilde{k}} = -\zeta_{\tilde{F}}$. On the other hand, using eq. (3.28) and the zero-energy condition plus the requirement of regularity in the IR, one gets a single solution $\zeta_{\tilde{f}-\tilde{k}} = X \cosh\tau$. The solutions for $\zeta_{\tilde{f}}$, $\zeta_{\tilde{k}}$, $\zeta_{\tilde{F}}$, ζ_y, ζ_ϕ, are [11]

$$\begin{aligned}
\zeta_{\tilde{f}} &= \frac{1}{2}X(\cosh\tau + 1)\,, \qquad \zeta_{\tilde{k}} = \frac{1}{2}X(-\cosh\tau + 1)\,, \qquad \zeta_{\tilde{F}} = -X\sinh\tau\,, \\
\zeta_y &= 2PX(\tau\,\cosh\tau - \sinh\tau)\,, \qquad \dot{\zeta}_\phi = 0\,.
\end{aligned} \tag{3.33}$$

Moreover, from the equation for $\phi(\tau)$ there is a unique regular solution for $\tau \to 0$, that corresponds to $\zeta_\phi = 0$. This implies that $\bar{\phi}$ is a constant. In the case of $\bar{y}$ the solution is

$$\bar{y}(\tau) = 64\,P\,X\,2^{2/3}\,\epsilon^{-8/3}\,\frac{1}{\sinh\tau}\int_0^\tau \frac{x\,\coth x - 1}{(\sinh(2x) - 2x)^{2/3}}\,\sinh^2 x\,dx\,. \tag{3.34}$$

Using this solution, the corresponding one for $\bar{p}(\tau)$ is formally given by the integral

$$\bar{p}(\tau) = \frac{1}{5\,\beta(\tau)}\int_0^\tau \frac{\bar{y}(x)}{\sinh x}\,\beta(x)\,dx\,, \tag{3.35}$$

where

$$\beta(x) = e^{2\int_{\tau_0}^{\tau}\exp[-10p_0(x)]\,dx}\,. \tag{3.36}$$

In the case of $\bar{Y}(\tau)$ and $\bar{q}(\tau)$ it is convenient to solve the first order differential equation for their difference $\bar{Y}(\tau) - \bar{q}(\tau)$, and it gives

$$\bar{Y}(\tau) - \bar{q}(\tau) = \int_\tau^\infty \left(\frac{1}{5}\frac{\bar{y}(x)}{\sinh x} + \frac{4}{3}\,e^{-10p_0(x)}\,\bar{p}(x)\right)\,dx\,. \tag{3.37}$$

Still, it remains to solve the equations involving the three-form field strengths. From eqs. (3.29), (3.30) and (3.31), and recalling the definitions of the $\tilde{f}$, $\tilde{k}$ and $\tilde{F}$ in terms of f, k and F, respectively, there is the following first order ODE system

$$\begin{aligned}
\dot{\bar{f}} + \dot{\bar{k}} + 2\,\sinh(2y_0(\tau))\,\bar{F}(\tau) - 2\,\bar{y}(\tau)\left(\dot{f}_0(\tau) - \dot{k}_0(\tau)\right) &= 2\frac{X}{P}h_0(\tau)\,, \\
\dot{\bar{F}}\,\coth\tau - \dot{\bar{f}} - \frac{\coth\tau}{2}\,(\bar{k} - \bar{f}) - e^{2y_0(\tau)}\,\bar{F} + 2\,\dot{f}_0(\tau)\,\bar{y}(\tau) &= -\frac{X}{P}h_0(\tau)\,, \\
\dot{\bar{F}}\,\coth\tau + \dot{\bar{k}} - \frac{\coth\tau}{2}\,(\bar{k} - \bar{f}) - e^{-2y_0(\tau)}\,\bar{F} + 2\,\dot{k}_0(\tau)\,\bar{y}(\tau) &= \frac{X}{P}h_0(\tau)\,,
\end{aligned} \tag{3.38}$$

where $h_0(\tau)$ is defined in appendix B. In addition, there is the following first order ODE for a linear combination of Y and q derivatives

$$\begin{aligned}
-2\dot{\bar{Y}} + 5\dot{\bar{q}} = \sqrt{3}\,e^{-4p_0(\tau)-6q_0(\tau)}\;\times& \\
\times\left(-(4\bar{p}(\tau) + 6\bar{q}(\tau))\,\tilde{L}_0 + (2P - \tilde{F}_0)\,\bar{\tilde{f}} + \tilde{F}_0\,\bar{\tilde{k}} + (\tilde{k}_0 - \tilde{f}_0)\,\bar{\tilde{F}}\right)&.
\end{aligned} \tag{3.39}$$

The formal expressions for $\bar{f}$, $\bar{k}$, $\bar{F}$ and $\bar{q}$ were obtained in [11] and we quote them in appendix C.

Now, we can study in detail the perturbation ansatz for the 3-form and 5-form field strengths when we consider the previously studied non-supersymmetric deformation of the KS background. Using the ansatz (3.8) for δF_3 we obtain

$$\begin{aligned} *\delta F_3 \;=\; & \frac{1}{18\sqrt{3}}\, e^{-q(\tau)-4p(\tau)+4Y(\tau)} \times \\ & \times [f_1\, e^{15q(\tau)-6Y(\tau)}\, da \wedge d\tau \wedge g^1 \wedge g^2 \wedge g^3 \wedge g^4 \wedge g^5 - \\ & \quad - 36\, f_2(\tau)\, e^{-q(\tau)-4p(\tau)-2Y(\tau)}\, (*_4 da) \wedge d\tau \wedge dg^5 \wedge g^5 + \\ & \quad + 81\, \dot{f}_2(\tau)\, e^{-q(\tau)+16p(\tau)-2Y(\tau)}\, (*_4 da) \wedge g^1 \wedge g^2 \wedge g^3 \wedge g^4] , \end{aligned} \tag{3.40}$$

which becomes the perturbation (2.6) when the metric (3.1) reduces to the Klebanov-Strassler background by turning off the non-supersymmetric deformation. Similarly, using the perturbation ansatz for the five-form field strength

$$\delta F_5 = (1 + *)\, \delta F_3 \wedge B_2 , \tag{3.41}$$

together with the expression for $H_3 = dB_2$

$$\begin{aligned} H_3 \;=\; -\Big[& d\tau \wedge (\dot{\tilde{f}}(\tau)\, g^1 \wedge g^2 + \dot{\tilde{k}}(\tau)\, g^3 \wedge g^4) + \\ & \frac{1}{2}\, (\tilde{k}(\tau) - \tilde{f}(\tau))\, g^5 \wedge (g^1 \wedge g^3 + g^2 \wedge g^4) \Big] , \end{aligned} \tag{3.42}$$

we obtain

$$\delta F_5 \wedge H_3 = f_1 \frac{(g_s\, M\, \alpha')^2}{4}\, \frac{d(f(\tau)\, k(\tau))}{d\tau}\, (*_4 da) \wedge g^1 \wedge g^2 \wedge g^3 \wedge g^4 . \tag{3.43}$$

In order to solve the remaining equation $d(*\delta F_3) = \delta F_5 \wedge H_3$ we can rewrite the perturbation of the five-form field strength (3.41) as

$$\delta F_5 = (1 + *)\, \delta \mathcal{F}_5 , \tag{3.44}$$

where

$$\delta \mathcal{F}_5 = \delta F_3 \wedge B_2 , \tag{3.45}$$

while for δF_3 we can write

$$\delta F_3 = f_1\, (*_4 da) + \Delta \delta F_3 , \tag{3.46}$$

where

$$\Delta \delta F_3 = -d(f_2(\tau)\, da \wedge g^5) , \tag{3.47}$$

is an exact form. Therefore, we get

$$\delta F_5 \wedge H_3 = (1 + *)\, f_1\, (*_4 da) \wedge B_2 \wedge H_3 , \tag{3.48}$$

where we have used the fact that $dg^5 \wedge B_2$ and $d\tau \wedge g^5 \wedge H_3$ vanish. Then, $d(*\delta F_3) = \delta F_5 \wedge H_3$ gives the following second order ODE for the fluctuation $f_2(\tau)$ in the non-supersymmetric deformation of the KS solution

$$-\frac{d}{d\tau}\Big(\dot{f}_2(\tau) e^{-2q(\tau)+12p(\tau)+2Y(\tau)}\Big) + \frac{8}{9} f_2(\tau) e^{-2q(\tau)-8p(\tau)+2Y(\tau)} = \frac{f_1}{12\sqrt{3}} (g_s M \alpha')^2 \frac{d(f(\tau) k(\tau))}{d\tau} . \tag{3.49}$$

As expected, for $\tilde{\delta}=0$ this equation reduces to eq. (2.9). In order to solve this equation for f_2 we consider the expansion for all the fields once the non-supersymmetric deformation is turned on. For f_2 we explicitly write

$$f_2(\tau) = f_2^0(\tau) + \tilde{\delta}\cdot \bar{f}_2(\tau) + \mathcal{O}(\tilde{\delta}^2)\,, \tag{3.50}$$

where $f_2^0(\tau)$ is the corresponding solution (2.17) obtained by Gubser, Herzog and Klebanov in the supersymmetric case that we have reviewed in the previous section. We can solve eq. (3.49) by considering the first order perturbation expansion for the exponential factors

$$\begin{aligned} e^{-2q(\tau)+12p(\tau)+2Y(\tau)} &= \epsilon^{4/3}\, K^4(\tau)\, \sinh^2\tau\, \times \\ &\times \left(1+\tilde{\delta}\,[-2\bar{q}(\tau)+12\bar{p}(\tau)+2\bar{Y}(\tau)]\right) + \mathcal{O}(\tilde{\delta}^2)\,, \qquad (3.51)\\ e^{-2q(\tau)-8p(\tau)+2Y(\tau)} &= \frac{\epsilon^{4/3}}{K^2(\tau)}\, \times \\ &\times \left(1+\tilde{\delta}\,[-2\bar{q}(\tau)-8\bar{p}(\tau)+2\bar{Y}(\tau)]\right) + \mathcal{O}(\tilde{\delta}^2)\,, \qquad (3.52)\end{aligned}$$

where we have replaced q_0, p_0 and Y_0 for their explicit functions corresponding to the KS background. Thus, at zero order in $\tilde{\delta}$ we recover eq. (2.9). Therefore, we only need to solve the corresponding equation for $\bar{f}_2(\tau)$ as follows

$$\begin{aligned} 0 = &-\frac{d}{d\tau}\left(K^4(\tau)\, \sinh^2\tau\, (\dot{\bar{f}}_2(\tau) + f_2^0(\tau)\,[-2\,\bar{q}(\tau)+12\bar{p}(\tau)+2\bar{Y}(\tau)])\right) + \qquad (3.53)\\ &+\frac{8}{9}\,\frac{1}{K^2(\tau)}\,\left(\bar{f}_2(\tau)+f_2^0(\tau)\,[-2\,\bar{q}(\tau)-8\bar{p}(\tau)+2\bar{Y}(\tau)]\right) - \frac{(g_sM\alpha')^2}{3\epsilon^{4/3}}\,\frac{d(f_0\,\bar{k}+k_0\,\bar{f})}{d\tau}\,.\end{aligned}$$

Note that the homogeneous differential equation for $\bar{f}_2(\tau)$ is exactly the same as it was for $f_2^0(\tau)$, i.e.

$$-\frac{d}{d\tau}[\dot{\bar{f}}_2(\tau)\,K^4(\tau)\,\sinh^2\tau] + \frac{8\,\bar{f}_2(\tau)}{9\,K^2(\tau)} = 0\,. \tag{3.54}$$

As in the supersymmetric case this is solved by the functions

$$\bar{f}_2^{(1)}(\tau) = [\sinh(2\tau)-2\tau]^{1/3}\,, \tag{3.55}$$

$$\bar{f}_2^{(2)}(\tau) = [\sinh(2\tau)-2\tau]^{-2/3}\,. \tag{3.56}$$

To solve the inhomogeneous equation we use the wronskian

$$W(\bar{f}_2^{(1)}(\tau), \bar{f}_2^{(2)}(\tau)) = -\frac{4\sinh^2\tau}{[\sinh(2\tau)-2\tau]^{4/3}}\,. \tag{3.57}$$

Then, we recast eq. (3.53)

$$\frac{d^2\bar{f}_2(\tau)}{d\tau^2} + \left(\frac{1}{K^4(\tau)\sinh^2\tau}\frac{d}{d\tau}(K^4(\tau)\,\sinh^2\tau)\right)\frac{d\bar{f}_2(\tau)}{d\tau} - \frac{8}{9\,K^6(\tau)\sinh^2\tau}\bar{f}_2(\tau) = \bar{j}(\tau)\,, \tag{3.58}$$

such that now the source becomes

$$\begin{aligned}\bar{j}(\tau) = &\left[-\eta\,\frac{d}{d\tau}(f_0(\tau)\bar{k}(\tau)+k_0(\tau)\bar{f}(\tau)) + \frac{8}{9\,K^2(\tau)}f_2^0(\tau)\,[-2\bar{q}(\tau)-8\bar{p}(\tau)+2\bar{Y}(\tau)] - \right.\\ &\left. -\frac{d}{d\tau}\left(K^4(\tau)\sinh^2\tau\, f_2^0(\tau)\,[-2\bar{q}(\tau)+12\bar{p}(\tau)+2\bar{Y}(\tau)]\right)\right]\frac{1}{K^4\,\sinh^2\tau}\,, \qquad (3.59)\end{aligned}$$

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where $\eta = \frac{(g_s M\alpha')^2}{3\epsilon^{4/3}}$. The general solution is

$$\bar{f}_2(\tau) = d_1 \cdot \bar{f}_2^{(1)}(\tau) + d_2 \cdot \bar{f}_2^{(2)}(\tau) - \bar{f}_2^{(1)}(\tau) \int_0^\tau \bar{f}_2^{(2)}(x) \frac{\bar{j}(x)}{W(\bar{f}_2^{(1)}(x), \bar{f}_2^{(2)}(x))} dx + \\ + \bar{f}_2^{(2)}(\tau) \int_0^\tau \bar{f}_2^{(1)}(x) \frac{\bar{j}(x)}{W(\bar{f}_2^{(1)}(x), \bar{f}_2^{(2)}(x))} dx , \tag{3.60}$$

where d_1 and d_2 are constants determined by the boundary conditions. In the present case in order to ensure the finiteness of the solution at $\tau \to 0$ we require $d_2 = 0$, while for $\tau \to \infty$ it implies that

$$d_1 = \frac{891}{32} \frac{(g_s M\alpha')^3}{2^{2/3}} \frac{X}{\epsilon^4} . \tag{3.61}$$

We are interested in analyzing the asymptotic behaviour of $f_2(\tau)$, both at zero and infinity, in order to determine whether the supergravity mode associated with the perturbation ansatz given by eqs. (3.7), (3.8) and (3.9) is normalizable.

Asymptotic behaviour of $f_2(\tau)$ for $\tau \to 0$. Let us consider the asymptotic behaviour for the previously studied fields when $\tau \to 0$. They lead to the following expressions

$$h_0(\tau) \approx 2^{2/3} \frac{(4P)^2}{\epsilon^{8/3}} (a_0 - a_1 \tau^2) + \mathcal{O}(\tau^3) , \tag{3.62}$$

$$\bar{y}(\tau) \approx \frac{3^{2/3}}{27} \mu \tau^2 + \mathcal{O}(\tau^4) , \tag{3.63}$$

$$\bar{p}(\tau) \approx \frac{3^{2/3}}{675} \mu \tau^2 + \mathcal{O}(\tau^4) , \tag{3.64}$$

$$\bar{q}(\tau) \approx \frac{3^{2/3} \cdot 11}{4050} \mu \tau^2 + \mathcal{O}(\tau^4) , \tag{3.65}$$

$$\bar{Y}(\tau) \approx C_Y^0 - \frac{3^{2/3}}{405} \mu \tau^2 + \mathcal{O}(\tau^4) , \tag{3.66}$$

$$\bar{F}(\tau) \approx \gamma \tau^2 + \mathcal{O}(\tau^4) , \tag{3.67}$$

$$\bar{f}(\tau) \approx \frac{1}{2} \gamma \tau^3 + \mathcal{O}(\tau^5) , \tag{3.68}$$

$$\bar{k}(\tau) \approx -2 \gamma \tau + \mathcal{O}(\tau^3) , \tag{3.69}$$

where a_0, a_1, C_Y^0 are constants, while

$$\mu = 96 \cdot 2^{1/3} g_s P X \epsilon^{-8/3} , \qquad \gamma = -\frac{2^{1/3}}{18} \mu a_0 . \tag{3.70}$$

As we have already seen from the supersymmetric case there is an integral expression for $f_2^0(\tau)$ which is well behaved for $\tau \to 0$. We explicitly obtain

$$f_2^0(\tau) = \frac{a_0 (g_s M\alpha')^2}{6 \cdot 3^{1/3} \epsilon^{4/3}} \tau - \frac{a_0 (g_s M\alpha')^2}{45 \cdot 3^{1/3} \epsilon^{4/3}} \tau^3 + \mathcal{O}(\tau^5) , \tag{3.71}$$

while for $\bar{f}_2(\tau)$ eq. (3.60) reduces to

$$\bar{f}_2(\tau) = \frac{(g_s M\alpha')^3 X}{\epsilon^4} \left(\frac{297 \cdot 3^{2/3}}{32} \tau + \left(\frac{3^{2/3} \cdot 99}{160} - \frac{2^{1/3} \cdot 4 a_0}{3^{2/3} \cdot 75} \right) \tau^3 \right) + \mathcal{O}(\tau^4) , \tag{3.72}$$

which in the low energy expansion for $f_2(\tau)$ falls with the same power of τ as the leading term of $f_2^0(\tau)$.

Asymptotic behaviour of $f_2(\tau)$ for $\tau \to \infty$. Now, we consider the opposite asymptotic limit of the fields, i.e. when $\tau \to \infty$. The corresponding expressions are

$$\begin{aligned}
h_0(\tau) &\approx 2^{1/3} \cdot \frac{3}{4}\, \alpha\, \tau e^{-4\tau/3} + \cdots\,, && (3.73)\\
\bar{y}(\tau) &\approx \mu\left(\tau - \frac{5}{2}\right) e^{-\tau/3} + V\, e^{-\tau} + \cdots\,, && (3.74)\\
\bar{p}(\tau) &\approx \frac{3}{5}\,\mu\,(\tau - 4)\, e^{-4\tau/3} + \cdots\,, && (3.75)\\
\bar{q}(\tau) &\approx -\frac{2}{5}\,\mu\,\tau\, e^{-4\tau/3} + \cdots\,, && (3.76)\\
\bar{Y}(\tau) &\approx \mu\left(\frac{1}{2}\tau - \frac{99}{40}\right) e^{-4\tau/3} + \cdots\,, && (3.77)\\
\bar{F}(\tau) &\approx 3\,\mu\left(\frac{1}{4}\tau - 1\right) e^{-\tau/3} + \left(\frac{3V}{2} + V'\right) e^{-\tau} + \mathcal{O}(e^{-4\tau/3})\,, && (3.78)\\
\bar{f}(\tau) &\approx -\frac{27}{16}\,\mu\, e^{-\tau/3} + \left(\frac{V}{2} + V'\right) e^{-\tau} + \mathcal{O}(e^{-4\tau/3})\,, && (3.79)\\
\bar{k}(\tau) &\approx \frac{27}{16}\,\mu\, e^{-\tau/3} - \left(\frac{V}{2} + V'\right) e^{-\tau} + \mathcal{O}(e^{-4\tau/3})\,, && (3.80)
\end{aligned}$$

where $\alpha = 4(g_s M \alpha')^2\, \epsilon^{-8/3}$, while V and V' are constants.

In addition, we have

$$f_2^0(\tau) = \frac{3(g_s M \alpha')^2}{4 \cdot 2^{1/3}\, \epsilon^{4/3}}\, \tau\, e^{-2\tau/3} + \mathcal{O}(\tau\, e^{-8\tau/3})\,. \tag{3.81}$$

In the limit $\tau \to \infty$ eq. (3.60) reduces to

$$\bar{f}_2(\tau) = \frac{891\,(g_s M \alpha')^3\, X}{\epsilon^4}\, e^{-4\tau/3} + \mathcal{O}(\tau^2\, e^{-2\tau})\,. \tag{3.82}$$

Therefore, we have explicitly shown that the supergravity mode associated with the perturbation ansatz is well-behaved both in the IR and UV limits.

Asymptotic behaviour of δF_3. Now, we study the normalizability of δF_3. As in the previous section we must integrate

$$\int \sqrt{|g|}\, |\delta F_3|^2\, d^{10}x = \int \delta F_3 \wedge *\delta F_3\,, \tag{3.83}$$

using the deformed solutions that we have obtained in this section. The above integral can be written as the sum of the following ones

$$\begin{aligned}
I_1^0 &= -f_1^2\, \frac{\epsilon^4}{96} \int_0^\infty h_0^2(\tau)\, \sinh^2\tau\, d\tau && (3.84)\\
\bar{I}_1 &= -f_1^2\, \frac{\epsilon^4}{96} \int_0^\infty h_0^2(\tau)\, \sinh^2\tau\, [14\bar{q}(\tau) - 4\bar{p}(\tau) - 2\bar{Y}(\tau)]\, d\tau\,, && (3.85)
\end{aligned}$$

$$I_2^0 = \frac{\epsilon^{4/3}}{3} \int_0^\infty \frac{(f_2^0(\tau))^2}{K^2(\tau)}\, d\tau\,, \tag{3.86}$$

$$\bar{I}_2 = \frac{\epsilon^{4/3}}{3} \int_0^\infty \frac{1}{K^2(\tau)} \left((f_2^0(\tau))^2\, [-2\bar{q}(\tau) - 8\bar{p}(\tau) + 2\bar{Y}(\tau)] + 2 f_2^0(\tau)\, \bar{f}_2(\tau)\right)\, d\tau\,, \tag{3.87}$$

$$I_3^0 = \frac{3\,\epsilon^{4/3}}{8} \int_0^\infty (\dot{f}_2^0(\tau))^2\, K^4(\tau)\, \sinh^2\tau\, d\tau\,, \tag{3.88}$$

$$\begin{aligned} \bar{I}_3 = {} & \frac{3\,\epsilon^{4/3}}{8} \int_0^\infty K^4(\tau)\, \sinh^2\tau\, \times \\ & \times \left((\dot{f}_2^0(\tau))^2\, [-2\bar{q}(\tau) + 12\bar{p}(\tau) + 2\bar{Y}(\tau)] + 2\dot{f}_2^0(\tau)\, \dot{\bar{f}}_2(\tau)\right)\, d\tau\,. \end{aligned} \tag{3.89}$$

We have written the integrals I_i^0 and $\bar{I}_i$ above according to the expansion $I_i = I_i^0 + \tilde{\delta} \cdot \bar{I}_i + \mathcal{O}(\tilde{\delta}^2)$, for $i = 1$, 2 and 3. These integrals are well-behaved in the IR limit. All the integrals I_i^0 fall as $e^{-2\tau/3}$ in the UV limit (when τ becomes large), while $\bar{I}_i$'s fall even faster, as $e^{-2\tau}$, in the UV limit.

This shows that we have found a normalizable zero-mode for the non-supersymmetric deformation of the warped deformed conifold. This is associated with a massless pseudo-scalar glueball in the non-supersymmetric gauge theory. We will return to this result in the next section.

It is a trivial check to show how to get the supersymmetric perturbation obtained by Gubser, Herzog and Klebanov reviewed in the previous section by just taking the non-supersymmetric deformation parameter $\tilde{\delta} = 0$. This, of course, sets the situation back to the supersymmetric Klebanov-Strassler background since $\tilde{\delta} = 0$ means that the solutions to the second order equations of motion satisfy the first order ones. In such a case $f_2(\tau)$ becomes $f_2^0(\tau)$, being well-behaved both at zero and infinity, and the perturbation to the three-form field strength is also normalizable.

4. Zero modes, glueballs and symmetry breaking in supersymmetric and non-supersymmetric dual gauge theories

In this section we study properties of the dual field theory associated with the axionic non-supersymmetric deformation that we have discussed in the previous section, and perform a parallel analysis to the case discussed by Gubser, Herzog and Klebanov.

Since our solution is based on the one reported in [11], it is a regular, non-supersymmetric first order deformation of the KS solution which has its same isometries. Thus, in our solution the deformation corresponds to the inclusion of a mass term of the gaugino bilinears in the dual field theory. The KS background has a well-known dual description in terms a four dimensional $\mathcal{N} = 1$ supersymmetric Yang Mills theory with the $\mathrm{SU}(N+M) \times \mathrm{SU}(N)$ gauge group, where on the supergravity side N and M are the numbers of regular and fractional D3-branes, respectively. This gauge theory is coupled to four bifundamental chiral multiplets A_i and B_j, with i and $j = 1, 2$ transforming under $\mathrm{SU}(N+M) \times \mathrm{SU}(N)$ gauge group. In addition, each set of fields A_i and B_j transforms as a doublet under the action of one of the two SU(2)'s in the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ global symmetry group. The theory is believed to undergo a cascade of Seiberg dualities towards the IR. As pointed out in [1], the cascade stops leading to the $\mathrm{SU}(2M) \times \mathrm{SU}(M)$ gauge group. The KS background has a deformation parameter $a_0 \sim \epsilon^{-8/3}$, such that it is related to a four dimensional mass scale $m \sim \epsilon^{2/3}$. For a non-vanishing ϵ the $\mathrm{U}(1)_R$ symmetry of the conifold is broken down to $\mathbb{Z}_2$, which is preserved by the gaugino bilinear $\mathrm{Tr}\,\lambda\,\lambda$. In this case the potential that appears in the effective action has a critical point corresponding to the superconformal theory dual to the $AdS_5 \times T^{1,1}$ background, where there are no fractional D3-branes. Now, if one expands the potential around the critical point and uses the usual mass/dimension formula of the AdS/CFT correspondence, one gets the dimension of the fields that can be identified with certain gauge theory operators [21, 22]. In particular, two of these operators are $\mathrm{Tr}(W_{(1)}^2) - W_{(2)}^2)$ (which on the supergravity side corresponds to the mode $y(\tau)$), and $\mathrm{Tr}(W_{(1)}^2) + W_{(2)}^2)$, (associated with $\zeta_2(\tau) \sim -F(\tau) + (k(\tau) - f(\tau))/2)$, having both dimension $\Delta = 3$. The non-supersymmetric deformation studied by Kuperstein and Sonnenschein was obtained by introducing mass terms of the gaugino bilinears associated with ζ_2 and y. The most general gaugino bilinear has the form of $\mu_+ \mathcal{O}_+ + \mu_- \mathcal{O}_- +$ c.c., where $\mathcal{O}_\pm \sim \mathrm{Tr}(W_{(1)}^2) \pm W_{(2)}^2)$, and $W_{(i)}$ with $i = 1, 2$ labels the $\mathrm{SU}(N+M)$ and $\mathrm{SU}(N)$ gauge groups, respectively. The deformation [11] has only one real parameter μ. All the fields behave regularly in the IR and in the UV limits, however, as mentioned before there are two non-normalizable modes which are precisely $y(\tau)$ and $\zeta_2(\tau)$. The mass term induces the so-called soft supersymmetry breaking. One should notice that, as was pointed out in [11], when in the deformed solution (C.1) the constants are chosen as $C_1 = -1/2$ while $C_2 = C_3 = 0$, a $(0,3)$ form is obtained (see appendix C). This breaks the supersymmetry and the solution diverges at $\tau \to \infty$. On the other hand, the difference between the vacuum energy of the deformed non-supersymmetric theory and the corresponding one of the supersymmetric Klebanov-Strassler solution is finite.

In the previous section we have obtained a normalizable zero mode in a non-supersymmetric deformation of the Klebanov-Strassler background. The presence of such a zero mode is related to the properties of the non-supersymmetric deformation inherited from the KS solution, and it is related to a spontaneously broken symmetry. Particularly, as in the KS case we would like to argue that in the dual field theory this massless mode is identified with a massless pseudo-scalar glueball generated by the spontaneously broken $\mathrm{U}(1)_B$ baryon number symmetry.[2] An important point here is the fact that in the framework of the AdS/CFT duality global symmetries in the field theory become gauge symmetries in the dual supergravity theory. Now, in order to identify the gauge field on the field theory description with the massless pseudo-scalar fluctuation in the type IIB supergravity description the argument given in [1] does straightforwardly apply. Firstly, let us remember that in the case of the dual superconformal fixed point of the conifold background solution [26] the global symmetries are $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)_R \times \mathrm{U}(1)_B$, which are all preserved. In particular, in [27, 21] the gauge field A that is dual to the baryon number current J^μ was identified as $\delta C_4 \sim \omega_3 \wedge A$.

[2] We should emphasize that apart from the massless glueballs found in [1], in previous calculations [23, 24, 25] of glueball spectra no massless modes were found.

On the other hand, when we consider the Klebanov-Strassler solution the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ global symmetry is preserved, while the global $\mathrm{U}(1)_R$ symmetry group breaks down to $\mathbb{Z}_{2M}$ in the UV, due to the chiral anomaly. This $\mathbb{Z}_{2M}$ symmetry spontaneously breaks down to $\mathbb{Z}_2$. There is no Goldstone boson associated with this symmetry breaking.

The axionic perturbation is a massless speudo-scalar glueball interpreted as the Goldstone boson of spontaneously $\mathrm{U}(1)_B$ baryon number symmetry. From the structure of the five-form field strength perturbation in eq. (2.3) one can read off the relation between the supergravity zero-mode and the glueball, since in the UV limit there is a component in δF_5 which behaves as $\omega_3 \wedge da \wedge d\tau$, leading to the identification of gauge field A with da. In the non-supersymmetric deformation the same comments apply, and in both cases the effective four-dimensional lagrangian is

$$\frac{1}{f_a}\int d^4x\, J^\mu(x)\,\partial_\mu a(x) = -\frac{1}{f_a}\int d^4x\, a(x)\,(\partial_\mu\, J^\mu(x))\,. \tag{4.1}$$

Particularly, in the non-supersymmetric deformation we can easily identify the supergravity mode associated with the $\mathrm{U}(1)_B$, just looking at the UV limit of the eq. (3.9), where we also find a component with structure like $\omega_3 \wedge da \wedge d\tau$, that can be used to identify the gauge field A with da.

The theory is on the baryonic branch since the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ global symmetry is preserved [4, 6, 1], and the only operators which preserve such a symmetry are the baryonic operators, so that they do have non-vanishing VEVs. In the supersymmetric field theory the series of Seiberg dualities stops at the theory with gauge group $\mathrm{SU}(2M)\times\mathrm{SU}(M)$, which is coupled to bifundamental fields A_i and B_j, $i,\ j = 1,\ 2$. The $\mathrm{SU}(2M)\times\mathrm{SU}(M)$ gauge invariant baryonic operators in the field theory are

$$\mathcal{B} \approx \epsilon_{\alpha_1\alpha_2\cdots\alpha_{2M}}\,(A_1)_1^{\alpha_1}(A_1)_2^{\alpha_2}\cdots(A_1)_M^{\alpha_M}(A_2)_1^{\alpha_{M+1}}(A_2)_2^{\alpha_{M+2}}\cdots(A_2)_M^{\alpha_{2M}}\,, \tag{4.2}$$

$$\bar{\mathcal{B}} \approx \epsilon^{\alpha_1\alpha_2\cdots\alpha_{2M}}\,(B_1)_{\alpha_1}^1(B_1)_{\alpha_2}^2\cdots(B_1)_{\alpha_M}^M(B_2)_{\alpha_{M+1}}^1(B_2)_{\alpha_{M+2}}^2\cdots(B_2)_{\alpha_{2M}}^M\,. \tag{4.3}$$

This theory has a superpotential, such that the baryonic branch is one of the supersymmetric background solutions where $\mathrm{SU}(2)\times\mathrm{SU}(2)$ global symmetry is preserved. It leads to identify the dual of this background with the baryonic branch of the cascading theory. On the other hand, the expectation values of the baryonic operators spontaneously break the $\mathrm{U}(1)_B$ baryon number symmetry. The vacuum corresponding to the deformed conifold has VEVs for the baryonic operators such as $|\mathcal{B}| = |\bar{\mathcal{B}}| = \Lambda_{2M}^{2M}$. Essentially, the baryonic branch can be parametrized by ξ, such that $\mathcal{B} = i\xi\Lambda_{2M}^{2M}$ and $\bar{\mathcal{B}} = i\xi^{-1}\Lambda_{2M}^{2M}$. Therefore, the pseudo-scalar Goldstone boson must correspond to change ξ by a phase. This is because the baryon number symmetry transforms the bifundamentals as $A_i \to e^{i\alpha}A_i$ and $B_j \to e^{-i\alpha}B_j$. Then, $f_a\partial_\mu a(x)$ is created through the action of the axial baryon number current J_μ on the vacuum. Precisely, $f_a\partial_\mu a(x)$ is the gradient of the pseudo-scalar Goldstone boson.

In the supersymmetric gauge theory it is expected to find a massless scalar field, the saxion, which was explicitly identified with its dual supergravity fluctuation mode in [1]. In addition, as remarked in [1] the original $\mathbb{Z}_2$ symmetry related to the interchange of the two S^2's in the base of the singular conifold is preserved by the warped deformed conifold metric, as well as the F_5, while F_3 and H_3 change sign. In the dual gauge theory it leads to the interchange of the two doublets of bifundamentals. The axion, both in the supersymmetric and non-supersymmetric cases, breaks this $\mathbb{Z}_2$ symmetry. We can see this from the perturbation ansatze (2.3) and (3.9), respectively.

It would be interesting to extend these studies to non-supersymmetric deformations of different backgrounds. For instance, the solution obtained by Pando Zayas and Tseytlin [14] has the same asymptotic behaviour as the KS one. However, this solution deals with a resolved conifold instead of the deformed one. In [28] it was pointed out that this solution is not supersymmetric. This solution is singular and has a repulson. It would be possible to resolve such a singularity through an enhancon mechanism, replacing it by some distribution of branes consistent with this background. On the other hand, as it was proposed in [6] it might be possible that after resolving the singularity, the solution becomes supersymmetric. In any case, an interesting point is that the field theory is conjectured to be on a mesonic branch of the moduli space, where the mesonic operators would break the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ global symmetry. Therefore, it would certainly be of interest to identify the supergravity modes associated with the Goldstone bosons related to the spontaneously broken $\mathrm{SU}(2)\times\mathrm{SU}(2)$ global symmetry. The fact that in this case the 2-cycle does not collapse may suggest that the $\mathrm{U}(1)_B$ baryon number symmetry would remain unbroken. These are matters of conjecture and deserve to be carefully studied.

5. Discussion and conclusions

We have obtained an explicit solution for an axionic perturbation of the non-supersymmetric deformation of the warped deformed conifold of Kuperstein-Sonnenschein. The corresponding supergravity fluctuation is normalizable, and we have performed an analysis analogous to the one developed by Gubser, Herzog and Klebanov in the supersymmetric warped deformed conifold. This is the Goldstone boson associated with the spontaneous breaking of the $\mathrm{U}(1)_B$ baryon number symmetry in the dual gauge theory of the non-supersymmetric warped deformed conifold background. Moreover, the gauge theory is on the baryonic branch of the moduli space since the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ symmetry of the dual supergravity background is preserved, and therefore the only operators with non-vanishing vacuum expectation values are the baryonic ones. Still, there are some points that deserve further investigation. It was argued that the stability of the non-supersymmetric solution requires a mass gap in the original dual supersymmetric gauge theory [29]. On the other hand, Gubser, Herzog and Klebanov have shown that in the Klebanov-Strassler solution there is no mass gap. Therefore, a very important question arises, i.e. whether the non-supersymmetric deformation of Kuperstein and Sonnenschein induces tachyonic modes, as well as if the saxion becomes a tachyonic mode itself. In addition, it would be useful to discuss the stability of the D1-branes at the bottom of the non-supersymmetric background. It would be expected that they were stable for small values of the non-supersymmetric deformation parameter.

On the other hand, although in the Maldacena-Núñez solution the integral $\int \delta F_3 \wedge * \delta F_3$ has a divergence that may be related to the linear dilaton background associated with the D5-brane in the UV, it would be interesting both, to find a normalizability criterion for

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the supergravity mode associated with the axion field perturbation, as well as to extend the analysis in the lines proposed here for non-supersymmetric deformations of this solution [29]–[32]. As suggested in [1], perhaps it may be possible to understand D-strings in the IR of the solution [5] as axionic strings.

It would also be interesting to obtain explicitly the supergravity fluctuation modes associated with the would-be mesonic branch of the moduli space related to the dual gauge theory of the Pando Zayas-Tseytlin background.

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A. Type IIB supergravity in ten dimensions

In order to define our notation we briefly review the action and equations of motion corresponding to ten dimensional type IIB supergravity [33]. The field content of this theory is given by the metric, a 4-form potential C_4, a scalar ϕ, an axion χ, a R-R 2-form potential C_2, a NS-NS 2-form potential B_2, two gravitinos with the same chirality Ψ^i_M, and two dilatinos λ_i ($i = 1, 2$). Since there is not a simple covariant lagrangian for type IIB supergravity under the condition $F_5 = *F_5$, one can write a lagrangian without constraining the five-form field strength and, after derivation of the equations of motion, one can impose that condition [34]. We use the notation given in [11] (see also [35]). We consider the bosonic type IIB supergravity action written in the Einstein frame

$$\begin{aligned}
S_{\rm IIB} = &\frac{1}{2\,\kappa^2}\int d^{10}x\,\sqrt{-g}\,R_E - \frac{1}{4\,\kappa^2}\times \\
&\times\int d^{10}x\,\Big[d\phi\wedge *d\phi + e^{2\phi}\,d\chi\wedge *d\chi + g_s\,e^{-\phi}\,H_3\wedge *H_3 + g_s\,e^{\phi}\,F_3\wedge *F_3 + \\
&\qquad + \frac{g_s^2}{2}\,F_5\wedge *F_5 + g_s^2\,C_4\wedge H_3\wedge F_3\Big],
\end{aligned} \tag{A.1}$$

with usual definitions for the fields. The equations of motion derived from the action (A.1) are

$$\begin{aligned}
R_{MN} = &\frac{1}{2}\,\partial_M\phi\,\partial_N\phi + \frac{1}{2}\,e^{2\phi}\,\partial_M\chi\,\partial_N\chi + \frac{g_s^2}{96}\,F_{MPQRS}\,F_N{}^{PQRS} + \\
&+\frac{g_s}{4}\,e^{\phi}\left(F_{MRS}\,F_N{}^{RS} - \frac{1}{12}\,F_{RST}\,F^{RST}\,g_{MN}\right) + \\
&+\frac{g_s}{4}\,e^{-\phi}\left(H_{MRS}\,H_N{}^{RS} - \frac{1}{12}\,H_{RST}\,H^{RST}\,g_{MN}\right),
\end{aligned} \tag{A.2}$$

$$d*d\phi = e^{2\phi}\,d\chi\wedge *d\chi + \frac{g_s\,e^{\phi}}{2}\,F_3\wedge *F_3 - \frac{g_s e^{-\phi}}{2}\,H_3\wedge *H_3\,, \tag{A.3}$$

$$d(e^{2\phi} * d\chi) = -g_s\,e^{\phi}\,H_3\wedge *F_3\,, \tag{A.4}$$

$$d(e^{\phi} * F_3) = g_s F_5 \wedge H_3 \,, \tag{A.5}$$
$$d * (e^{-\phi} H_3 - e^{\phi} \chi F_3) = -g_s F_5 \wedge F_3 \,. \tag{A.6}$$

In addition, there are the following Bianchi identities

$$dF_3 = -d\chi \wedge H_3 \,, \tag{A.7}$$
$$dF_5 = H_3 \wedge F_3 \,, \tag{A.8}$$
$$dH_3 = 0 \,. \tag{A.9}$$

In this paper the ten dimensional axion χ of type IIB supergravity is set to zero, while we call axionic perturbation to a perturbation included in the fluctuation spectrum of type IIB supergravity defined as a particular combination of perturbations of the three-form and five-form field strengths (see sections 2 and 3).

B. A collection of formulas of the Klebanov-Strassler solution

We write some explicit formulas of [4] which are relevant for the calculations we have presented. We have used the notation given in [1], [15] and [16]. The 10d metric is

$$ds_{10}^2 = h_0(\tau)^{-1/2} (-dt^2 + d\vec{x}_3^2) + h_0(\tau)^{1/2} ds_6^2 \,, \tag{B.1}$$

where the warped deformed conifold metric is given by

$$\begin{aligned} ds_6^2 = & \frac{\epsilon^{4/3}}{2} K(\tau) \Bigg[\frac{1}{3K(\tau)^3} (d\tau^2 + (g^5)^2) + \\ & + \cosh^2\left(\frac{\tau}{2}\right) ((g^3)^2 + (g^4)^2) + \sinh^2\left(\frac{\tau}{2}\right) ((g^1)^2 + (g^2)^2) \Bigg] \,, \end{aligned} \tag{B.2}$$

where

$$K(\tau) = \frac{(\sinh(2\tau) - 2\tau)^{1/3}}{2^{1/3} \sinh\tau} \,, \tag{B.3}$$
$$h_0(\tau) = (g_s M \alpha')^2 \, 2^{2/3} \, \epsilon^{-8/3} \, I(\tau) \,, \tag{B.4}$$
$$I(\tau) = \int_\tau^\infty dx \, \frac{x \coth x - 1}{\sinh^2 x} (\sinh(2x) - 2x)^{1/3} \,. \tag{B.5}$$

The one-forms are

$$\begin{aligned} g^1 &= \frac{e^1 - e^3}{\sqrt{2}} \,, \qquad g^2 = \frac{e^2 - e^4}{\sqrt{2}} \,, \\ g^3 &= \frac{e^1 + e^3}{\sqrt{2}} \,, \qquad g^4 = \frac{e^2 + e^4}{\sqrt{2}} \,, \\ e^5 &= g^5 \,, \end{aligned} \tag{B.6}$$

where

$$\begin{aligned} e^1 &\equiv -\sin\theta_1 \, d\phi_1 \,, \\ e^2 &\equiv d\theta_1 \,, \\ e^3 &\equiv \cos\psi \, \sin\theta_2 \, d\phi_2 - \sin\psi \, d\theta_2 \,, \\ e^4 &\equiv \sin\psi \, \sin\theta_2 \, d\phi_2 + \cos\psi \, d\theta_2 \,, \\ e^5 &\equiv d\psi + \cos\theta_1 \, d\phi_1 + \cos\theta_2 \, d\phi_2 \,. \end{aligned} \tag{B.7}$$

The NS-NS two form field is

$$B_2 = \frac{g_s M \alpha'}{2} \left[f(\tau)\, g^1 \wedge g^2 + k(\tau)\, g^3 \wedge g^4 \right], \tag{B.8}$$

while its corresponding three form field strength is given by

$$\begin{aligned} H_3 = dB_2 = \frac{g_s M \alpha'}{2} \Big[& d\tau \wedge (\dot{f}(\tau)\, g^1 \wedge g^2 + \dot{k}(\tau)\, g^3 \wedge g^4) + \\ & + \frac{1}{2} (k(\tau) - f(\tau))\, g^5 \wedge (g^1 \wedge g^3 + g^2 \wedge g^4) \Big] . \end{aligned} \tag{B.9}$$

The R-R three form field strength is

$$\begin{aligned} F_3 = \frac{M \alpha'}{2} \Big[& (1 - F(\tau))\, g^5 \wedge g^3 \wedge g^4 + F(\tau)\, g^5 \wedge g^1 \wedge g^2 + \\ & + \dot{F}(\tau)\, d\tau \wedge (g^1 \wedge g^3 + g^2 \wedge g^4) \Big] . \end{aligned} \tag{B.10}$$

In addition

$$F(\tau) = \frac{\sinh\tau - \tau}{2 \sinh\tau} , \tag{B.11}$$

$$f(\tau) = \frac{\tau \coth\tau - 1}{2 \sinh\tau} (\cosh\tau - 1) , \tag{B.12}$$

$$k(\tau) = \frac{\tau \coth\tau - 1}{2 \sinh\tau} (\cosh\tau + 1) , \tag{B.13}$$

where they correspond to the first term in each of the equations in (C.1). Also, we introduce the definition two-form

$$\omega_2 = \frac{1}{2} (g^1 \wedge g^2 + g^3 \wedge g^4) , \qquad \omega_3 = g^5 \wedge \omega_2 . \tag{B.14}$$

Notice that

$$\frac{d\,(k(\tau)\, f(\tau))}{d\tau} = \frac{1}{2} (\tau \coth\tau - 1) \left(\coth\tau - \frac{\tau}{\sinh^2\tau} \right) . \tag{B.15}$$

To calculate Hodge dual of the forms it is useful to have

$$\sqrt{|g|} = \frac{\sqrt{h_0(\tau)}\, \epsilon^4 \sinh^2\tau}{96} , \tag{B.16}$$

where $|g|$ is the determinant of the metric (B.1).

Certain identities are useful for the explicit checks of the EOM. Indeed, they are

$$2\,\omega_2 \wedge \omega_2 = g^1 \wedge g^2 \wedge g^3 \wedge g^4 , \tag{B.17}$$

$$dg^5 = -(g^1 \wedge g^4 + g^3 \wedge g^2) , \tag{B.18}$$

$$dg^5 \wedge dg^5 = -4\,\omega_2 \wedge \omega_2 , \tag{B.19}$$

$$H_3 \wedge d\tau \wedge g^5 = 0 , \tag{B.20}$$

$$H_3 \wedge dg^5 = 0 . \tag{B.21}$$

C. Kuperstein-Sonnenschein explicit solutions

Here we introduce the solutions of the ODE system of eqs. (3.38) and (3.39) obtained by Kuperstein and Sonnenschein.

First notice that solving the type IIB supergravity EOM, it is found a first order ODE system for $F(\tau)$, $f(\tau)$ and $k(\tau)$, from where the following solutions result [11]

$$\begin{aligned} F(\tau) &= \frac{1}{2} - \frac{\tau}{2\sinh\tau} + C_1\left(\cosh\tau - \frac{\tau}{\sinh\tau}\right) + C_2\,\frac{1}{\sinh\tau}\,, \\ f(\tau) &= \frac{\tau\,\coth\tau - 1}{2\sinh\tau}\,(\cosh\tau - 1) + \\ &\quad + C_1\left(2\tau - \sinh\tau - \tanh\frac{\tau}{2} - \frac{\tau}{2\cosh^2\frac{\tau}{2}}\right) + C_2\,\frac{1}{2\cosh^2\frac{\tau}{2}} + C_3\,, \\ k(\tau) &= \frac{\tau\,\coth\tau - 1}{2\sinh\tau}\,(\cosh\tau + 1) + \\ &\quad + C_1\left(2\tau + \sinh\tau - \coth\frac{\tau}{2} + \frac{\tau}{2\sinh^2\frac{\tau}{2}}\right) - C_2\,\frac{1}{2\sinh^2\frac{\tau}{2}} + C_3\,. \end{aligned} \tag{C.1}$$

Now, for eqs. (3.38) and (3.39) the solutions can be written as

$$\bar f(\tau) = \lambda_1(\tau)\,f_1(\tau) + \lambda_2(\tau)\,f_2(\tau) + \lambda_3(\tau)\,f_3(\tau)\,, \tag{C.2}$$

$$\bar k(\tau) = \lambda_1(\tau)\,k_1(\tau) + \lambda_2(\tau)\,k_2(\tau) + \lambda_3(\tau)\,k_3(\tau)\,, \tag{C.3}$$

$$\bar F(\tau) = \lambda_1(\tau)\,F_1(\tau) + \lambda_2(\tau)\,F_2(\tau) + \lambda_3(\tau)\,F_3(\tau)\,, \tag{C.4}$$

where $f_i(\tau)$, $k_i(\tau)$ and $F_i(\tau)$, for $i = 1$, 2 and 3 are the functions in eqs. (C.1) above that appear multiplied by the constants C_i's, respectively. The λ_i's [11] are

$$\lambda_1(\tau) = \frac{1}{2}\int_\tau^\infty \frac{k_0'(x) + f_0'(x)}{\sinh x}\,\bar y(x)\,dx\,, \tag{C.5}$$

$$\lambda_2(\tau) = \frac{1}{2}\int_0^\tau \left(\frac{k_0'(x) + f_0'(x)}{2}\,\bar y(x)\,\left(\cosh x - \frac{x}{\sinh x}\right) - \frac{X}{P}\,h_0(x)\,\sinh^2 x\right) dx\,, \tag{C.6}$$

$$\lambda_3(\tau) = -\int_\tau^\infty \left((f_0'(x) - k_0'(x))\,\bar y(x) - \frac{1}{2}\sum_{i=1}^{2}\left((k_i(x) + f_i(x))\lambda_i'(x)\right) + \frac{X}{P}\,h_0(x)\right) dx\,, \tag{C.7}$$

while $\bar q$ is given by

$$\begin{aligned} \bar q(\tau) = \frac{1}{\rho(\tau)}\int_0^\tau \rho(x)\Bigg[&\frac{2}{3}(\dot{\bar Y} - \dot{\bar q}) + \frac{1}{\sqrt 3}\,e^{-4p_0(x) - 6q_0(x)}\,\tilde L_0(x)\,\times \\ &\times\left(-4\bar p(x) + \frac{1 - F_0(x)}{L_0(x)}\bar f(x) + \frac{F_0(x)}{L_0(x)}\bar k(x) + \frac{k_0(x) - f_0(x)}{L_0(x)}\bar F(x)\right)\Bigg] dx\,, \end{aligned} \tag{C.8}$$

where

$$\rho(\tau) = \exp\left[2\sqrt3\int_{\tau_0}^\tau e^{-4p_0(x) - 6q_0(x)}\,\tilde L_0(x)\,dx\right]. \tag{C.9}$$

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